Calculation of a weak nonleptonic matrix element using "Weinberg" sum rules

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Abstract

There is a "toy" weak matrix element which can be expressed as an integral over the vector and axial vector spectral functions, $\rho_V(s) - \rho_A(s)$. I review our recent evaluation of these spectral functions, the study of four "Weinberg" sum rules and the calculation of this matrix element. [Talk presented at the XXVIII International Conference on High Energy Physics, ICHEP94, Glasgow, Aug. 1994, to be published in the proceedings.]

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Weak nonleptonic matrix elements are notoriously difficult to calculate by any reliable method. E. Golowich and I have proposed a novel weak matrix element, not found in the standard model, which can be well calculated by a mixture of theoretical and phenomenological inputs[1]. In the process we had to update the status of the Weinberg sum rules[2]. This talk briefly reviews these developments.

Consider a weak matrix element formed using only vector currents

$$\tilde{\mathcal{H}}_w = \frac{g^2}{8}\int d^4x D_F(x, M_w) T\left(\bar{d}(x)\gamma_\mu u(x)\bar{u}(0)\gamma^\mu s(0)\right). \tag{1}$$

Aside from KM factors, this differs from the weak Hamiltonian of the Standard Model only in that the latter involves left handed (V-A) currents. However under chiral symmetry the Hamiltonian in Eq. 1 has a different transformation property since $8_V = 8_L \oplus 8_R$, where V, L, R refer to transformations under vectorial, left handed and right handed SU(3) respectively. The $8 \oplus 8$ direct product contains an $(8_L, 8_R)$ term

$$\begin{aligned} 8 \otimes 8 &= (8_L, 8_R) + (8_L, 1_R) \\ &+ (1_L, 8_R) + (27_L, 1_R) + (1_L, 27_R). \end{aligned} \tag{2}$$

The $(8_L, 8_R)$ component is special because it is the only term which does not vanish in the chiral limit $(m_q \to 0, p \to 0)$. The value of a $K \to \pi$ matrix element in the chiral limit (which we adopt hence forth) is then calculable using the soft pion theorem to remove the pseudoscalars

$$\begin{aligned} &\langle \pi(p) \mid \tilde{\mathcal{H}}_w \mid K(p)\rangle \\ &\quad = \frac{-g^2}{F_\pi^2}\int d^4x D_F(x, M_w) \\ &\qquad\quad \langle 0 \mid T\left(V_\mu(x)V^\mu(0) - A_\mu(x)A^\mu(0)\right) \mid 0\rangle \\ &\quad = \frac{-g^2}{F_\pi^2}\int d^4x D_F(x, M_w)\left[\pi_V(x) - \pi_A(x)\right] \end{aligned} \tag{3}$$

where $\pi_{V,A}$ are the vector and axial polarization tensors. After writing these in terms of the spectral densities, one obtains

$$\langle \pi(p) \mid \tilde{\mathcal{H}}_w \mid K(p) \rangle = \frac{G_F}{\sqrt{2}} A \tag{4}$$

where

$$A = M_w^2 \int ds \frac{s^2}{s - M_w^2} ln \left(s/M_w^2 \right) \left[\rho_V(s) - \rho_A(s) \right] \tag{5}$$

This is similar to four other sum rules.

$$\begin{aligned}
\int \frac{ds}{s} \left[\rho_V(s) - \rho_A(s) \right] &= -4\bar{L}_{10} \\
\int ds \left[\rho_V(s) - \rho_A(s) \right] &= F_\pi^2 \\
\int ds \; s \left[\rho_V(s) - \rho_A(s) \right] &= 0 \\
\int ds \; s \; ln(s) \left[\rho_V(s) - \rho_A(s) \right] & \\
&= -\frac{16\pi^2 F_\pi^2}{3e^2} \left(m_\pi^2 - m_\pi^2 \right)
\end{aligned} \tag{6}$$

valid in the chiral limit. Here the second and third of these are the original two Weinberg sum rules[3]. The first sum rule above involves the chiral coefficient $\bar{L}_{10} = -(9.1 \pm 0.3) \times 10^{-3}$ which is measured in radiative pion decay. This sum rule originates in work of Das et al and was given in its present, more general, form by Gasser and Leutwyler[4]. The final sum rule comes from the calculation of the electromagnetic contribution to the mass difference of neutral and charged pions[5] at lowest order in the chiral expansion. Although these were first derived before QCD, they rely on assumptions about the short distance properties that can only be proven through the use of QCD in the chiral limit. Note that the last two sum rules are no longer valid if the quark masses are turned on. This set of sum rules represents a beautiful interplay of the chiral and short distance properties of QCD.

The spectral functions can be constructed fairly reliably. This is not the place to discuss all aspects [see Ref. 2], but the low energy portion is known from chiral symmetry and the high energy effects are small and amenable

to treatment by perturbative QCD. The intermediate energy contributions are not theoretically calculable at present, but fortunately these may be extracted from e^+e and τ decay data. The vector spectral function starts out with two-pion and four-pion contributions, and relatively quickly approaches a constant value. The axial spectral function has three and five pion contributions and approaches the same constant. The difference between them goes to zero as s^{-3}, which vanishes so rapidly that there is not much contribution to the sum rules from high energy. There are minor uncertainties in the data, and we adjust the spectral functions within the range of experimental uncertainties in order to fit the data while accommodating the four Weinberg sum rules. The resulting forms for ρ_V and ρ_A separately and for the difference are given in Ref 2. While this procedure does not prove the Weinberg sum rules are required by the data, they certainly are easily compatible with the set of experimental information. The fact that it is easy to satisfy the Weinberg sum rules within the constraints of theory and data is very nontrivial and is a credit to the complex theoretical ideas that went into their formulation.

When applied to the weak matrix element we obtain

$$\begin{aligned} A &= -0.062 \pm 0.017 GeV^6 \\ \langle \pi(p) \mid \tilde{\mathcal{H}}_w \mid K(p) \rangle &= 5.3 \times 10^{-7} GeV^2 \end{aligned} \tag{7}$$

In contrast, the “vacuum saturation” approximation would yield

$$A_{vac-sat} = -0.033 GeV^6 \tag{8}$$

and the real weak matrix element extracted from $K \to 2\pi$ using chiral symmetry

$$\frac{\langle \pi(p) \mid \mathcal{H}_w \mid K(p) \rangle}{\mid V_{ud} V_{us}^* \mid} = 1.7 \times 10^{-7} GeV^2 \tag{9}$$

We see a modest enhancement of the matrix element.

This calculation has not uncovered the mechanism for the $\Delta I = \frac{1}{2}$ rule, as the $(8_L, 8_R)$ operator automatically does not have the freedom to have a $\Delta I = \frac{1}{2}$ enhancement, requiring $A_{3/2} = \frac{2}{3} A_{1/2}$ always. However inspection of the details of the calculation does reveal a hint as to why it is so difficult to

calculate nonleptonic amplitudes. There is very little contribution from either the high or low energy ends, where theory is useful. Most of the strength comes from intermediate energies, which are generally not under theoretical control. While we cannot apply this matrix element to Standard Model phenomenology, it should prove possible to use it as a test of lattice calculational methods. In addition, it is possible that this calculational technique may be extended to study more realistic matrix elements.

References

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